NASA TECHNICAL TRANSLATION NASA TT F-15,812

APPLICATION OF MATHEMATICAL STATISTICS METHODS TO THE STUDY OF ARTIFICIAL EARTH SATELLITE MOVEMENT NEAR THE CENTER OF MASS

N. F. Martynova

Translation of "Primeneniye metodov matematicheskoy statistiki k izucheniyu dvizheniya iskusstvennogo sputnika Zemli okolo tsentra mass," in: Mekhanika upravlyayemogo dvizheniya i problemy kosmicheskoy dinamiki, Ed. by V. S. Novoselov, Leningrad University Press, Leningrad, 1972, pp. 83-103.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D. C. 20546 AUGUST 1974

STANDARD TITLE PAGE

1. Report No. NASA TT F-15812	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle APPLICATION OF MATHEMATICAL STATISTICS METHODS TO THE STUDY OF ARTIFICIAL EARTH SATELLITE MOVEMENT NEAR THE CENTER OF MASS		5. Report Date August 1974
		6. Performing Organization Code
7. Author(s) N. F. Martynova		8. Performing Organization Report No.
		10. Work Unit No.
9. Performing Organization Name and Address Leo Kanner Associates Redwood City, California 94063		11. Contract or Grant No. NASw-2481-407
		13. Type of Report and Period Covered Translation
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration, Washington, D. C. 20546		14. Sponsoring Agency Code

15. Supplementary Notes

Translation of "Primeneniye metodov matematicheskoy statistiki k izucheniyu dvizheniya iskusstvennogo sputnika Zemli okolo tsentra mass," in: Mekhanika upravlyayemogo dvizheniya i problemy kosmicheskoy dinamiki, Leningrad University Press, Leningrad, 1972, pp. 83-103.

16. Abstract

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17. Key Words (Selected by Author(s))	18. Distribution Statement Unclassified-Unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	29	

APPLICATION OF MATHEMATICAL STATISTICS METHODS TO THE STUDY OF ARTIFICIAL EARTH SATELLITE MOVEMENT NEAR THE CENTER OF MASS

N. F. Martynova

The orientation of an artificial Earth satellite [AES] with /83 given shape and given dynamic characteristics depends on its position at an initial moment and on perturbing moments of external forces affecting the satellite. By assigning a certain model of these external forces, we can study orbital motion of an AES.But /84 due to diverse accidents, when the satellite separates from the booster rocket, the orientation and angular velocities of the satellite at the initial moment in time can be given only with an accuracy to within several deviations from the average values, i.e., parameters characterizing the orientation at the initial moment are quantities of a random nature, subordinate to some law of distribution.

In turn, the moments of external forces acting on a satellite are also functions of such random arguments as atmospheric density, Earth's magnetic field intensity, Solar radiation intensity, etc. Moreover, the satellite can be subjected to the perturbing effect of meteorites; consequently the angular velocities of its rotation about the center of mass can receive random increases at an arbitrary point in time.

According to currently available published materials, we do not have sufficiently complete information on the distributive laws of atmospheric density for different altitudes. Those empirical models of the atmosphere, which are employed at present, can be viewed only as models of average atmospheric density, without calculation of the spread of values of density around the average.

But in many studies ([2], [9], etc.) there is an indication that this dispersion can attain very large values according to the altitude and other factors.

D. G. King-Healey [2] notes that if on the Earth's surface deviations in density from its average reach 4%, then at altitudes from 160 to 240 km these deviations can reach 35%. At altitudes of 640-720 km, density can reach values which exceed the average by twice. Atmospheric density between 160 and 800 km is greatly affected by solar activity, and this effect is probable in nature.

The same can be said of the effect of Earth's magnetic field on the orientation of a satellite. According to [2], the intensity of Earth's magnetic field can be seen as the sum of two components:

1) the larger component varies slightly in time and its variations can be noted only over a large interval of time;

2) the second component, much less in amplitude, fluctuates greatly and these fluctuations are random by nature. The presence of this component is induced mainly by the existence of electrical currents in the upper atmosphere. Its magnitude is very sensitive to the intensity of solar radiation.

Therefore, the question arises on the study of the effect of the probable nature of all the above-numerated parameters on the nature of motion of an artificial satellite about the center of mass.

1. Equations of motion of AES about the center of mass

Let us consider the artificial satellite of Earth having a magnetic damper. The center of mass of the satellite moves in a circular orbit with an angular velocity of ω_0. With the orbit we will associate a rectangular system of coordinates $Cx_1y_1z_1$ with the origin in the center of mass of the satellite; moreover, we will examine a system of primary central axes of inertia of the satellite $Cxyz$. The orientation of the satellite in space will be characterized by angles associated with ordinarily used angles of Euler (Ψ_{Eu}, Θ_{Eu}, Φ_{Eu}) as follows:

$$\theta=\Psi_{Э}-\frac{\pi}{2}, \quad \varphi=\Theta_{Э}-\frac{\pi}{2}, \quad \psi=\Phi_{Э}.$$

Therefore, Ψ is the angle of intrinsic rotation, Θ is the angle of precession in the orbital plane read from the direction of the radius-vector of the satellite's center of mass, and Φ characterizes oscillations of the satellite's axis about the orbital plane.

The equations of motion of the AES in question about the center of mass in projections onto the primary central axes of inertia of the satellite will be the following:

$$\begin{aligned}
&I_x\dot{\omega}_x+\omega_y\omega_z(I_z-I_y)=L_x+K_x+M_x,\\
&I_y\dot{\omega}_y+\omega_x\omega_z(I_x-I_z)=L_y+K_y+M_y,\\
&I_z\dot{\omega}_z+\omega_x\omega_y(I_y-I_x)=L_z+K_z+M_z,\\
&\dot{\theta}=-\omega_0+\cos^{-1}\varphi\,(\omega_y\sin\psi+\omega_z\cos\psi),\\
&\dot{\varphi}=\omega_y\cos\psi-\omega_z\sin\psi,\\
&\dot{\psi}=\omega_x+\mathrm{tg}\,\varphi\,(\omega_y\sin\psi+\omega_z\cos\psi),\\
&\dot{\gamma}=\frac{B}{k_{\text{п}}}\,(H_x\cos\gamma\cos\Pi+H_y\cos\gamma\sin\Pi-H_z\sin\gamma),\\
&\dot{\Pi}=\frac{B}{k_{\text{п}}}\sin^{-1}\gamma\,(H_y\cos\Pi-H_x\sin\Pi).
\end{aligned} \tag{1}$$

Here two equations relative to the angles γ and Π, which describe the position of the vector of magnetic moment $\overline{B}$ of the core of the dampger with respect to the axes Cxyz are added to the first six equations of Euler relative to angles of orientation and projection of angular velocity of rotation on an axis rigidly connected to the satellite; L_x, L_y, L_z are projections of the moment of gravitational forces on the main axes of inertia; M_x, M_y, M_z are projections of the moment of perturbing forces; K_x, K_y, K_z are projections of the moment of effect on the body of the satellite on the part of the stabilizer; H_x, H_y, H_z are projections of magnetic field intensity (Earth's); B--the magnitude of the magnetic moment of the damper core; k_d--the damping coefficient of the satellite.

In the capacity of perturbing moments were investigated the moment of aerodynamic forces and the moment induced by the interaction of Earth's magnetic field with the satellite. The system of equations (1) can be reduced to the form /86

$$\boxed{\frac{dX_i}{dt}=Y_i(X_j,\ a_k,\ t),\quad i,\ j=1,\ \ldots,\ 8.}$$

The right sides of these equations are functions of time t, random arguments a_k and variables of motion. Accordingly, the variables of motion X_i, definable by this system (including angles of orientation and projection of angular velocity of AES rotation about the center of mass) are also functions of time and random arguments.

2. Equations for Elements of a Covariation Matrix of Motion

Due to the random nature of parameters of the initial position and the condition of the external medium, each realization of motion will differ from motion definable by average values of the parameters. All possible realizations of motion form a certain n-dimensional trajectory band around "average" motion. The width of this band is defined by a correlation matrix of motion, constructed as a time-function, i.e., by a vectorial moment of the II order. The structure of this band is defined by moments of higher orders. If at each moment in time the components of motion are distributed according to a normal law and the law of variation in time of the average vector of motion and the covariation matrix are given, the width and structure of the band of dispersion of motion is defined fully. In general, the question of studying the structure of the motion dispersion band is a complex question and can only be eludicated by some characteristics of this structure.

We should underscore that we are not investigating random functions, but functions of random arguments. In study [3] it is shown that the mathematical expectation of the derivative with respect to time of a function of random arguments and time is equal to the derivative of its mathematical expectation:

$$M[Y_i(t)] = \frac{d}{dt} M[X_i(t)],$$

where M--operator of mathematical expectation.

Apparently the following equation is valid:

$$\frac{d}{dt}(X_i - X_i^{\text{cp}}) = Y_i - Y_i^{\text{cp}},$$

where X^{av}_i and Y^{av}_i, for the sake of convenience, denote the mathematical expectations of the functions $X_i(t)$ and $Y_i(t)$.

Let us make a series of formal transformations: /87

On the other hand,

$$M\left\{\frac{d}{dt}[(X_i-X_i^{\text{ср}})\cdot(X_j-X_j^{\text{ср}})]\right\}=\frac{d}{dt}M[(X_i-X_i^{\text{ср}})(X_j-X_j^{\text{ср}})].$$

С другой стороны,

$$M\left\{\frac{d}{dt}[(X_i-X_i^{\text{ср}})(X_j-X_j^{\text{ср}})]\right\}=M\left[(X_i-X_i^{\text{ср}})\frac{d}{dt}(X_j-X_j^{\text{ср}})+\right.$$
$$\left.+(X_j-X_j^{\text{ср}})\cdot\frac{d}{dt}(X_i-X_i^{\text{ср}})\right]=M[(X_i-X_i^{\text{ср}})\cdot(Y_j-Y_j^{\text{ср}})+$$
$$+(X_j-X_j^{\text{ср}})\cdot(Y_i-Y_i^{\text{ср}})].$$

Therefore, to change elements of the covariation matrix K(t) of the process $\vec{X}(t)$ in time, the following system of equations must be derived:

$$\frac{d}{dt}K[X_i(t),\ X_j(t)]=K[X_i(t),\ Y_j(t)]+K[X_j(t),\ Y_i(t)]. \qquad (2)$$

The right sides of this system are functions of time and parameters of distribution of the probability density of random arguments of motion. For an n-dimensional process $\vec{X}(t)$, the order of this system will be equal to $\frac{n(n+1)}{2}$. For further analysis, it is useful to examine the case where each of the components of the random process can be written in the form

$$X_i(t)=A_i(t)+B_i\cos\omega_i t+C_i\sin\omega_i(t),$$

where A_i, B_i and C_i are functions of random parameters of motion with non-zero dispersions. The elements of the covariation matrix of motion in this case are expressed so:

$$K_{ij}(t)=M\{[(A_i-A_i^{\text{ср}})+(B_i-B_i^{\text{ср}})\cos\omega_i t+(C_i-C_i^{\text{ср}})\sin\omega_i t]\times$$
$$\times[(A_j-A_j^{\text{ср}})+(B_j-B_j^{\text{ср}})\cos\omega_j t+(C_j-C_j^{\text{ср}})\sin\omega_j t]\}=$$
$$=K(A_i,\ A_j)+K(B_i,\ A_j)\cos\omega_i t+K(C_i,\ A_j)\sin\omega_i t+$$
$$+K(A_i,\ B_j)\cdot\cos\omega_j t+K(A_i,\ C_j)\sin\omega_j t+$$
$$+\frac{1}{2}[K(B_i,\ B_j)-K(C_i,\ C_j)]\times\cos(\omega_i+\omega_j)t+$$
$$+\frac{1}{2}[K(C_i,\ B_j)+K(B_i,\ C_j)]\times\sin(\omega_i+\omega_j)t+$$
$$+\frac{1}{2}[K(B_i,\ B_j)+K(C_i,\ C_j)]\times\cos(\omega_i-\omega_j)t+$$
$$+\frac{1}{2}[K(C_i,\ B_j)-K(B_i,\ C_j)]\times\sin(\omega_i-\omega_j)t.$$

Therefore, if the components of the random process $X_i(t)$ and $X_j(t)$ contain harmonic components with frequencies of respectively ω_i and ω_j, the element of the covariation matrix will contain harmonic components with frequencies

$$\omega_i,\ \omega_j,\ \omega_i+\omega_j,\ |\omega_i-\omega_j|.$$

For variation in time of the dispersion of the component $X_i(t)$ /88 the following relation is valid:

$$D_i = D(A_i) + \frac{D(B_i)+D(C_i)}{2} + 2K(A_i,\ B_i)\cdot\cos\omega_i t +$$
$$+ 2K(A_i,\ C_i)\cdot\sin\omega_i t + \frac{D(B_i)-D(C_i)}{2}\cos 2\omega_i t + K(B_i,\ C_i)\cdot\sin 2\omega_i t.$$

Accordingly, if $X_i(t)$ has a harmonic component with the frequency ω_i, then the dispersion $D_i(t)$ will generally have components with frequencies ω_i and $2\omega_i$.

By knowing equations (1) of motion of an object, the distributive laws of random parameters of motion, we can try to construct a system of equations (2) for a covariation matrix of the process. But in general, this problem is rather unwieldy and a more productive means for solving the problem is the definition of the correlation characteristics of motion by the method of statistical tests (the Monte-Carlo method).

3. The Monte-Carlo Method

As we know [5], the Monte-Carlo method is based on simulation of a statistical experiment with the aid of a computer and the recording of numerical characteristics obtained from this experiment. To calculate estimates of average motion and the elements of the covariation matrix, the system of equations had to be integrated n times with random values of those parameters of motion whose effect was studied in the particular case; then the appropriate mathematical expectation and elements of the covariation matrix had to be averaged. To solve the problem using the Monte-Carlo method, we must have a subprogram to obtain random numbers distributed according to given laws with given characteristics. In the case in point, it was necessary to have subprograms of numbers distributed uniformly in some interval and numbers distributed in conformity with a normal law with given average values and dispersions.

System of equations (1) of motion of an artificial Earth satellite about the center of mass was integrated n times by the Runge-Kutta method in a specific interval of time in the course of the stated problem. At the start of calculation of each AES flight version, some random situation was fixed, i.e., by referring to the subprograms of random numbers, random values of the initial data of the system of differential equations of motion and the parameters of the state of the external medium were assigned. After checking n versions of motion and accumulating some information in the computer memory bank, estimates of the interesting probability characteristics of motion for fixed moments in time were defined in terms of formulas of mathematical /89
statistics. In particular:

1) for average values of components of motion

$$M^*[X_p(t_j)] = \frac{1}{n}\sum_{k=1}^{n} X_p^{(k)}(t_j),$$

2) for elements of the covariation matrix of motion

$$K[X_p(t_j),\ X_q(t_j)] = \frac{1}{n-1}\sum_{k=1}^{n}\{X_p^{(k)}(t_j) - M^*[X_p(t_j)]\}\times$$
$$\times\{X_q^{(k)}(t_j) - M^*[X_q(t_j)]\} =$$
$$= \frac{1}{n-1}\sum_{k=1}^{n} X_p^{(k)}(t_j)\cdot X_q^{(k)}(t_j) - \frac{n+1}{n-1} M^*[X_p(t_j)]\times M^*[X_q(t_j)].$$

As we know [4], these estimates for multidimensional normal laws are estimates of the greatest plausibility.

The number of n random realizations of AES motion about the center of mass, by which the statistical definition of estimates of mathematical expectation and elements of the covariation matrix of motion was made, was selected on the basis of requirements, the attainment of a satisfactory approximation of these estimates to their true values on one hand; and on the basis of limitations on computer time on the other hand. In evaluating the accuracy of the obtained characteristics, the Student law for mathematical expectation of motion and the law χ^2 with k degrees of freedom ($k = n - 1$) for elements of the covariation matrix was used. The maximally possible number of versions was considered to be $n = 50$. In some cases $n = 30$ was found to be sufficient.

4. The Effect of the Random Nature of the Parameters of State of the External Medium on the Orientation of an Artificial Satellite in Space

The effect of the random nature of the parameters of state of the external medium on the AES's orientation in space was investigated for the phase of oriented motion: dynamically symmetric satellite ($I_y = I_z$, $(I_x)/(I_y) = 3/40$) already captured by the gravitational field and only oscillations about the state of equilibrium are possible.

The initial data of the system of differential equations was given as follows: $\omega_x^0 = \omega_y^0 = 0$, $\omega_z^0 = 1.27\ \omega_0$, $\theta^0 = \phi^0 = \psi^0 = 0$, where ω_0 is orbital angular velocity. It was also considered that the density of the atmosphere was distributed in conformity with a normal law and can achieve values double those of the average. On this assumption was defined the density dispersion. It is necessary to note that according to [2], the distributive law of atmospheric density for altitudes on the order of 600-800 km /90 differs from the normal. But since the goal of this investigation is only the proof of the need for calculating the random nature of the parameters of state of the external medium in different problems on defining AES orientation in space, it suffices to examine the first approximation of the distributive law of atmospheric density in the form of a normal distributive law.

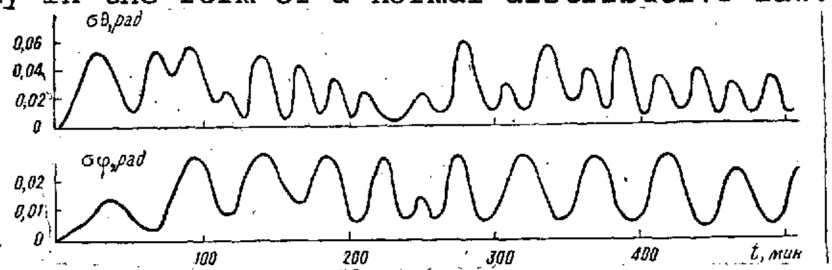

Fig. 1.

Fig. 1 illustrates variations in the mean quadratic deviations of angles θ and ϕ from the average values as a function of time. We should underscore that these deviations, induced by the presence of a dispersion of atmospheric density values, attain comparatively great amplitudes: for the angle θ the mean quadratic deviations reach up to 0.06 rad; for the angle ϕ --amplitudes of 0.03 rad. The average values of the angles of orientation coincide with the atmospheric density values. Consequently, $\sigma\theta$ and $\sigma\phi$ characterize direct perturbations in motion formed as a result of random deviations in atmospheric density from the average value.

Similar results have been obtained in simulation of AES motion about the center of mass under conditions of dispersed values of magnetic field intensity of the Earth around the values of some average model of intensity. As an average model of Earth's magnetic field, a dipole model was selected. For the dipole model of the magnetic field, projections of intensity on the axis of the orbital system of coordinates with a circular orbit are expressed as follows:

$$H^0_{x_1} = -0{,}63\left(\frac{R_3}{R_3+h}\right)^3 \sin i \sin \omega_0 t,$$
$$H^0_{y_1} = 0{,}315\left(\frac{R_3}{R_3+h}\right)^3 \sin i \cos \omega_0 t,$$
$$H^0_{z_1} = 0{,}315\left(\frac{R_3}{R_3+h}\right)^3 \cos i,$$

where h--altitude of the orbit above the Earth's surface; R_e--radius /91 of the Earth; i--orbital inclination. It was felt that projections of intensity of the actual Earth's magnetic field on the axis of an orbital system of coordinates can deviate from their average values by up to 15%. On that basis, dispersions of intensity projections H_{x1}, H_{y1}, H_{z1} were assigned. AES motion about the center of mass was investigated, as with the dispersion of values of atmospheric density, in the phase of oriented motion with the same initial data of the system of differential equations. Just as before, the average values of the angles of orientation coincide with the angles of orientation in nonperturbed motion, which proves the linear nature of motion. The mean quadratic deviations of the angles θ and ϕ from the averages reach values of 0.05 rad for the angle θ and 0.03 for the angle ϕ. Since in the phase of oriented motion the system of equations (1) for the model in point can be linearized with respect to all coordinates except for angle ψ, then under the supposition of independence of atmospheric and magnetic perturbations in the satellite's motion, the mean quadratic deviations of the angles of orientation θ and ϕ will be defined by the following equation:

$$\sigma = \sqrt{\sigma_a^2 + \sigma_m^2},$$

where σ_a^2 is the angular dispersion induced by the dispersion of values of atmospheric density, σ_m^2 is the angular dispersion induced by the dispersion of values of projections of Earth's magnetic field intensity.

Accordingly, only due to two aforementioned factors $\sigma\theta$ can attain values on the order of 0.078 rad, while $\sigma\theta$--0.042 rad. Therefore, in those problems where we must have great accuracy in knowing the satellite's orientation angles, we must take into account the random nature of the parameters of state of the external medium.

5. AES Motion After Separation from the Booster

According to [1], [7], [8], the booster carries the AES into orbit with great accuracy in the orientation angles. But at the moment of separation of the satellite from the booster, due to diverse accidents in the separation mechanism, the satellite can receive rather greater perturbations in angular velocity of rotation about the center of mass. It is worth evaluating the effect of these random perturbations on the nature of AES motion about the center of mass, the stability of motion, and on the capture time of the satellite by the gravitational field.

Under consideration--a dynamically symmetrical satellite. The booster lifts this satellite into a circular orbit about the Earth. At the moment of separation from the booster, the satellite is situated in space so that its axis of dynamic symmetry lies in the orbital plane and is directed along the transversal, i.e., the following values are the initial data of motion through the orientation angles: $_0 = {}_0 = 0,\ {}_0 = \frac{}{2}$. In the absence of perturbations of the initial state, the satellite is immediately turned by the gravitational field so that the axis of its dynamic symmetry is set along the radius-vector of the center of mass; the satellite, due to the presence of moments of perturbing force, makes small oscillations about the state of stable equilibrium of non-perturbed motion: /92

$$\psi^* = \varphi^* = \theta^* = 0$$

or

$$\psi^* = \varphi^* = 0, \quad \theta^* = \pi.$$

But, as has already been noted above, the actual picture of AES motion about the center of mass is much more complicated. The following model problem was investigated. At the moment of separation from the booster, the satellite receives some angular velocity of motion about the center of mass, wherein the values of the projections of this velocity onto the main central axes of inertia of the satellite are random quantities, uncorrelated and distributed in conformity to a normal law. The average values of projections of angular velocities were assigned according to an ideal picture of motion in the absence of initial perturbation:

$$(\omega_x^0)_{cp} = (\omega_y^0)_{cp} = 0, \qquad (\omega_z^0)_{cp} = \omega_0.$$

Dispersions of angular velocity projections were assigned so that

$$\sigma\omega_x = 6\omega_0,$$
$$\sigma\omega_y = \sigma\omega_z = 30\omega_0.$$

The initial angles of orientation ψ and ϕ were considered distributed according to a uniform law with the following average values:

$$\psi_0^{cp} = \varphi_0^{cp} = 0 \qquad \theta_0^{cp} = \frac{\pi}{2}.$$

Deviations in the angles ψ and ϕ from the average are small:

$$\max \Delta\psi_0 = \max \Delta\varphi_0 = 0{,}008 \ \textit{рад.}$$

Deviations in the angle of precession are negligibly small.

According to obtained results of simulation, the actual AES motion about the center of mass can sharply differ from unperturbed motion. The mean quadratic deviation of the angle θ (Fig. 2a) abruptly increases in time and after a time interval equal to 3 revolutions of the center of mass of the satellite around the Earth, reaches the value of about 500 rad. This proves that the predominant conditions for great initial perturbations in the angular rate of rotation are the conditions of rapid rotation of the axis of dynamic symmetry of the satellite around the normal /93
to the orbital plane (rapid precession of the satellite's axis).

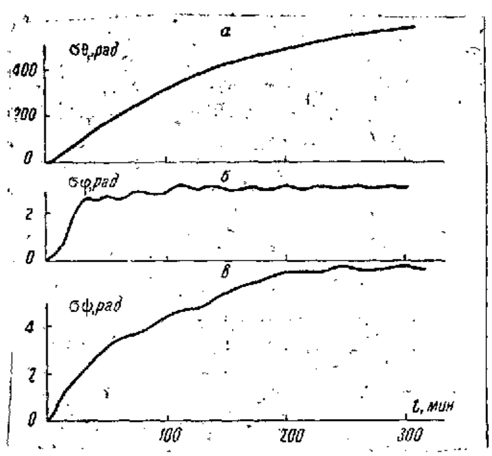

Fig. 2.

The width of the dispersion band of motion with respect to the angle of nutation (Fig. 2b) is much less: the mean quadratic deviation $\sigma\phi$ in the first turn of the center of mass of the satellite around the Earth reaches 3 rad and then maintains this constant value. Accordingly, even after the first turn of the center of mass of the satellite around the Earth, the nutation rotation of the axis of dynamic symmetry is replaced by conditions of oscillation (otherwise the quantity $\sigma\phi$ would continue to increase). For the angle of inherent satellite rotation, the bandwidth of dispersion reaches the value $\sigma\psi = 4.3$ rad even after the half-turn of the satellite's center of mass around the Earth and continues to increase, but already at a lower rate (Fig. 2c).

Figure 3 shows the nature of variation in mean quadratic deviations of projections of angular velocities. For all three projections there is a typical stable tendency to narrow to dispersion band. A comparative analysis was made on the velocity of narrowing of the dispersion band of x, y, $\dot{z}$ for a satellite with a magnetic damper and without a damper. In the developed program of simulation of this process, the charts were somewhat altered to exclude the effect of the magnetic damper on AES motion about the center of mass. In both modifications of the program, simulation was conduted by the Monte Carlo method of AES motion about the center of mass.

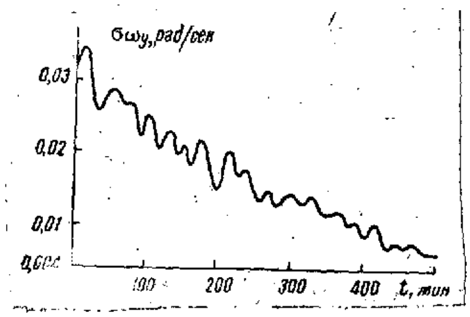

Fig. 3.

The initial data of the system of differential equations was assigned as follows:

$$\theta_0 = \frac{\pi}{2}, \quad \varphi_0 = \psi_0 = 0, \quad \omega_x^0 = \omega_y^0 = 0, \quad \omega_z^0 = \omega_0.$$

The dispersions of these quantities at the initial moment were considered such that /94

$$\sigma\omega_x^0 = 6\omega_0,$$
$$\sigma\omega_y^0 = \sigma\omega_z^0 = 30\omega_0.$$

After ten revolutions of the satellite's center of mass around the Earth, the dispersion of angular velocity values about the average changed as follows:

with a damper	without a damper
$\sigma\omega_x = 0{,}6\omega_0$	$\sigma\omega_x = 0{,}1\omega_0$
$\sigma\omega_y = \sigma\omega_z = 2{,}4\omega_0$	$\sigma\omega_y = \sigma\omega_z = 15\omega_0$

Therefore, these data make it possible to evaluate the efficiency of the applied damper: this magnetic damper increases the rate of extinction of precession and nutation oscillations of the axis of dynamic symmetry of the satellite. In this context, it also somewhat reduces the rate of stabilization of the satellite in terms of the angle of inherent rotation.

After 15 orbits of the Earth of the center of mass of the satellite with the magnetic damper $\sigma\omega_{xy}$ reaches the magnitudes

$$\sigma\omega_x = 0{,}4\omega_0, \qquad \sigma\omega_y = \sigma\omega_z = 1\omega_0.$$

Then the process of damping slows down: after 20 orbits

$$\sigma\omega_x = 0{,}3\omega_0, \qquad \sigma\omega_y = \sigma\omega_z = 0{,}7\omega_0$$

and the process of AES capture by the gravitational field begins.

6. Capture of the Satellite by the Gravitational Field

The process of transfer affected by gravitational field force of the Earth from the arbitrary motion of a satellite relative to the center of mass into an oscillatory motion about the state of stable equilibrium is called capture of the satellite by the gravitational field.

To illustrate the capture situation, one of the accidental realizations of motion was investigated with the following initial data:

$$\theta_0 = \frac{\pi}{2}, \quad \varphi_0 = \psi_0 = 0, \quad \omega_x^0 = \omega_y = 0, \quad \omega_z^0 = 2{,}9\omega_0.$$

In the absence of any perturbations of these initial data, motion of the axis of dynamic symmetry of the satellite is composed of nutation oscillations about the orbital plane of the center of mass, wherein the amplitude of these oscillations is small, on the order of 0.05 rad (Fig. 4). Precession of the axis around the normal to the plane of the orbit also turns into oscillatory motion about the direction of the radius-vector of the center of mass (Fig. 5). The angle of inherent rotation of

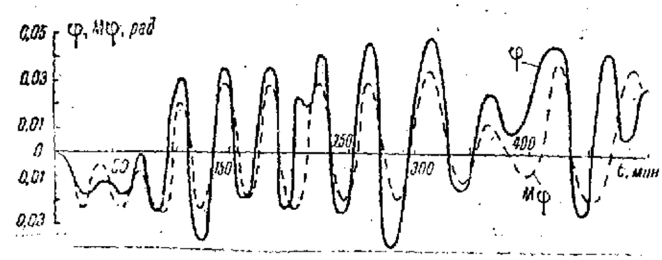

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Fig. 4.

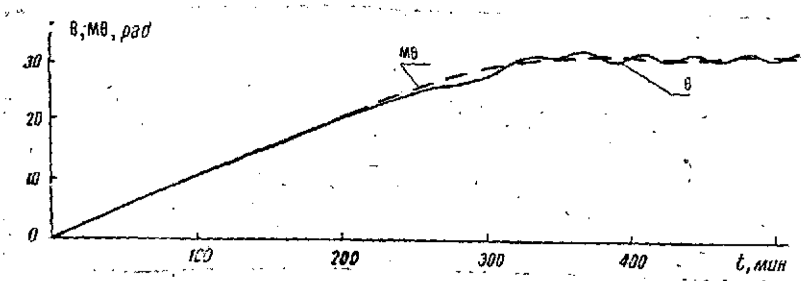

Fig. 5.

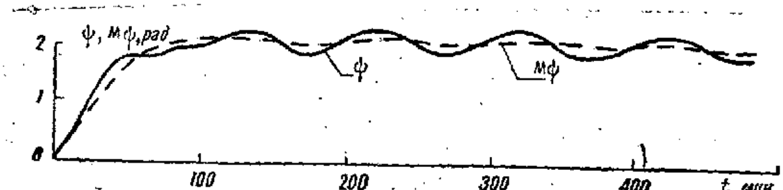

Fig. 6.

the satellite gradually ceases to increase, i.e., satellite rotation about the axis of dynamic symmetry attenuates (Fig. 6).

Let us now examine a certain tube of random trajectories around the previously examined trajectory. We will consider that at the initial moment of time, the width of this tube is defined by additional random perturbations of angular velocity of rotation with zero average value and with dispersions of projections such that

$$\sigma\omega_x^0 = \sigma\omega_y^0 = \sigma\omega_z^0 = 0{,}1\,\omega_0.$$

Let us consider the effect of these perturbations on the capture situation. As in the previous cases, definition of statistical characteristics was done in terms of 50 versions of motion. The average motion for these versions has the same nature as the aforenoted unique realization of motion. The mathematical expectation of the angle of nutation oscillations reaches the mag- /96

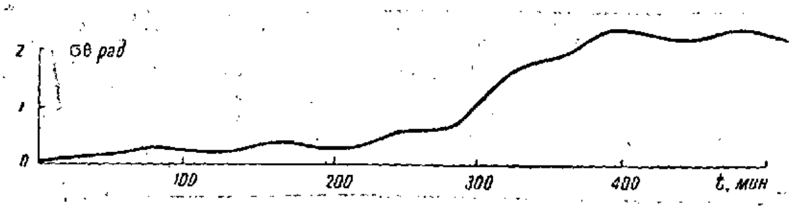

Fig. 7.

nitude of about 0.05 rad; but therein the magnitude of dispersion of this angle has the same order. The bandwidth of dispersion of the angles of inherent satellite rotation are established as constant and such that the mean quadratic deviation of angle ψ from its mathematical expectation do not exceed 0.5 rad. Accordingly, in all realizations of motion, the satellite transferred from conditions of rotation around the axis of dynamic symmetry into conditions of oscillation about some average value. But the conditions of capture are defined mainly by the nature of variation of angle θ. According to Figure 10, the mathematical expectation of angle θ increases from 1.5708 rad and then fluctuations are established $M\{\theta\}$ around the value θ = 31.4. The amplitude of these fluctuations is much less than the amplitude of fluctuations in the investigated particular realization of motion. The bandwidth of dispersion of motion in terms of angle θ, definable by $\sigma\theta$, is established as constant: $\sigma\theta \rightarrow 2.5$ rad (Fig. 7) which proves that in all 50 versions of motion occurred a stable capture of the satellite by the gravitational field. The verified derivation in print of different realizations of AES motion about the center of mass at the capture stage by the gravitational field showed that capture takes place about the value $\theta = \pi n$, where n--a natural number $n \in [8, 12]$, i.e., capture occurs namely about one of the states of stable equilibrium:

$$\varphi=0^\circ, \quad \theta=0^\circ,$$

or

$$\varphi=0^\circ, \quad \theta=180^\circ;$$

the value of the angle ψ, due to dynamic symmetry of the satellite, does not vary. It is only important that the satellite transferred to conditions of oscillation in terms of angle ψ.

The nature of variation of mathematical expectations of projections of angular velocity of rotation coincides with the nature of change of ω_x, ω_y, ω_z in this random realization of motion which here is motion with average values of initial data. The bandwidth of dispersion of ω_x, ω_y, ω_z about the averages is established as small and constant:

$$\sigma\omega_x \simeq 0.0001 \text{ rad/s},$$
$$\sigma\omega_y \simeq 0.0005 \text{ rad/s},$$
$$\sigma\omega_z \simeq 0.0006 \text{ rad/s}.$$

Therefore, as a result of capture of the satellite by the gravitational field, its motion acquires a qualitatively new nature. /97
We will call it the oriented motion of the AES about the center of mass.

7. Oriented Motion of a Satellite

In the phase of oriented motion, satellite oscillations around the state of stable equilibrium gradually attenuate and the amplitudes of the angles θ and ϕ become small. Perturbations of the orientation angles induced by the accidental nature of atmospheric density and the Earth's magnetic field intensity become comparable to the values of these angles in the absence of perturbations in the state of the external medium and consequently, must be taken into account in different problems associated with the definition of AES orientation. These perturbations were already examined in section 4 of this study. We also must explain the effect of random perturbations of initial data of motion. In the phase of oriented motion, we must convert from studying the covariation matrix of motion by the Monte Carlo method to the study of a system of differential equations for elements of the covariation matrix. This method allows us to conduct finer analysis of variation in the correlation characteristics of motion.

Let us consider a system of differential equations of AES motion about the center of mass in projections onto the axes of a semi-stationary system of coordinates in which the x axis is rigidly connected to the satellite's envelope and is an axis of dynamic symmetry; the y and z axes travel in a plane perpendicular to the x axis at a velocity of $\dot{\psi}$. The system of equations of motion for this satellite with a magnetic system of damping can be reduced to the following form after several transformations

$$\frac{d}{dt} X_i(t) = \sum_m a_{im}(t) \cdot X_m + c_i(t), \qquad i,\ m = 1, \ldots, 7,$$

where $a_{im}(t)$ and $c_i(t)$ are several time functions, parameters of state of the external medium and geometry of the satellite.

If we assume that the state of the external medium is not accidental, i.e., all parameters of state of the external medium are strictly defined in time, and the initial values of this system can tolerate deviations from the average values with some initial covariation matrix, then variation of this covariation matrix in time will, according to equations (2), be defined by the following system:

$$\frac{d}{dt}k_{ij}=\sum_m (a_{im}k_{mj}+a_{jm}k_{mi}). \tag{3}$$

(Order of this system is $N = \frac{n(n+1)}{2}$, i.e., where $n = 7$, $N = 28$). /98
We must note that in the case of linear motion, the law of distribution of probability density of initial data of motion does not function for variation in time of the elements of the covariation matrix of motion.

System (3) is a linear system of equations with variable coefficients. The system can be studied analytically; but since the system's order is quite high, it is more convenient to study it by computer.

To explain the effect on AES motion about the center of mass of the presence on board the satellite of a magnetic damper, system (3) was integrated with the same initial data both with allowance for moment of effect on the AES body from the magnetic damper and without it.

In solving this problem it was felt that at the initial moment of time, the satellite receives perturbation in angular velocity of rotation such that

$$\sigma^2\omega_x=\sigma^2\omega_y=\sigma^2\omega_z=0{,}001\,\omega_0^2,$$

i.e., the mean quadratic deviation of initial angular velocity in projections onto the axes of a semi-stationary system of coordinates constitutes

$$\sigma\omega_x = \sigma\omega_y = \sigma\omega_z = 0{,}033\omega_0,$$

where ω_0 is the orbital angular velocity of the satellite. All other elements of the covariation matrix are equal to zero at the initial moment.

Fig. 8 shows change in time of mean quadratic deviations $\sigma\omega_x$, $\sigma\omega_y = \sigma\omega_z$ and $\sigma\theta = \sigma\phi$ for an AES without a damper. In variation of dispersion $\sigma\omega_x$ there is present an harmonic component with a frequency of $2\omega_0$, i.e., in the process of motion of the satellite about the axis of dynamic symmetry may exist an harmonic component with a frequency of ω_0 and $2\omega_0$. The nature of variation of dispersions of angular velocities ω_y and ω_z, on one hand, and the orientation angles θ and ϕ on the other, is identical; but they are in opposite phase. We can also talk of the presence of an harmonic component in $\sigma_i(t)$; consequently, in motion itself; the frequency of this component in $\sigma_i(t)$ $\omega_1 = 5\omega_0$.

Standardized correlation matrix of AES motion without a damper is simple in form:

$$r_{\omega_y\varphi} \neq 0, \; r_{\omega_z\theta} \neq 0;$$

all other coefficients of the correlation are equal to zero.

Therefore, for motion in a circular orbit on the phase of oriented motion of a dynamically symmetrical AES without any sort of passive stabilization system, initial perturbation of angular velocity evokes stable non-attenuating perturbations both in angular velocity and in the orientation angles: a time-continuous average level of dispersion for the following components of mo- /99 tion is established

$$(\sigma\omega_y)_{cp} = (\sigma\omega_z)_{cp} = 0{,}01\omega_0,$$
$$(\sigma\theta)_{cp} = (\sigma\varphi)_{cp} = 0{,}011 \text{ рад},$$

and only perturbation of angular velocity in projection onto the axis of dynamic symmetry of the satellite gradually attenuates.

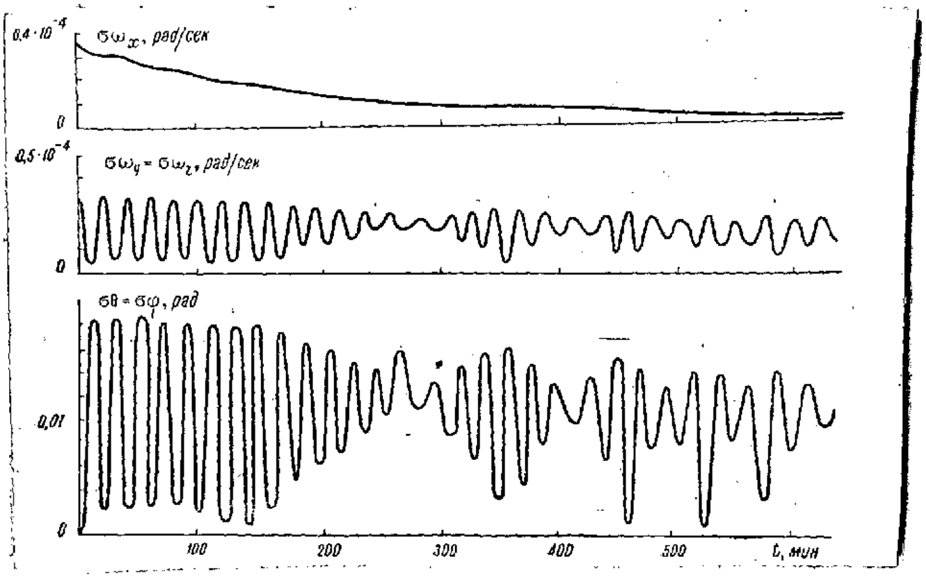

Fig. 8.

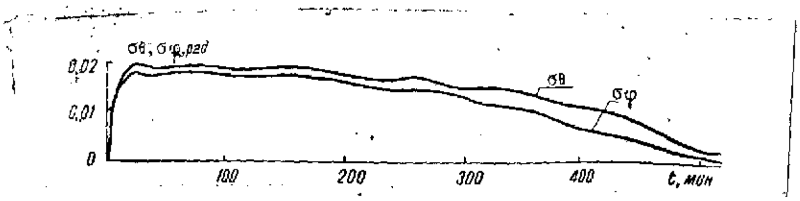

Fig. 9.

The situation is different if the satellite contains a magnetic damper system. In this event, according to Fig. 9, the dispersion of orientation angles quickly attenuates, i.e., the magnetic damper effectively counteracts different random perturbations of motion. In the dispersions of all components of motion, the presence of an harmonic component with a frequency equal to the frequency of orbital motion ω_0 is evident. The standardized correlation matrix of motion has a rather complex nature--all elements of this matrix depend on time and are greatly different from zero. In the variation of these elements (Fig. 10) the presence of a periodic component with a frequency of ω_0 is evident. /100

Therefore, the presence of a magnetic damper system on the AES (passive stabilization) sharply alters the probability nature of response in motion of the satellite around the center of mass to accidental perturbations of initial angular velocity of rotation: correlation connections between all components become real, coefficients of correlation vary in time, which greatly complicates the covariation analysis of motion. But the presence of an on-board magnetic passive stabilization system actively facilitates maintenance of stable motion and suppression of random perturbations.

We would also note that similar results for the phase of oriented motion of an AES about the center of mass were obtained even with the Monte Carlo method both for an AES with a magnetic damper and without one.

Moreover, the Monte Carlo method permitted us to solve the problem of the possible approximation of the distributive law of components of motion of an AES about the center of mass by the normal distributive law.

8. The Distributive Law of Components of AES Motion about the Center of Mass

In solving problems of defining AES orientation using probability methods, it is important to know to which type the process in point is related or, of equal value, what is the law of evolution of the process.

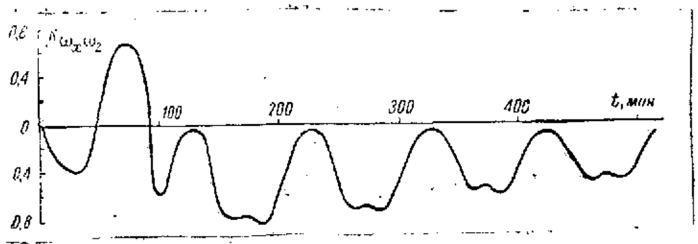

Fig. 10.

Among all known laws of distribution, particular place is occupied by the normal law of distribution, since, first of all, the normal law is fully defined by the assignment of averages and a covariation matrix; and secondly, most random quantities with which we must encounter in practice, are normally distributed.

In our case we are examining the function of random arguments /101
--the vectorial function; it is advisable to verify whether or not this process is a multidimensional Gaussian process.

We know that if a multidimensional law of distribution of a random vector is normal, then unidimensional laws of distribution of the components of this vector are also Gaussian. Consequently, to establish the nature of the law of distribution of the random vector in point, we must first establish how close are the unidimensional laws of distribution of the orientation angles and angular velocities to the normal law of distribution.

Let us write the law of distribution of the component X_i in a Gram-Charlier series [3]

$$f[X_i]=f_0[X_i]\cdot\left[1+\sum_{n=3}^{\infty}\frac{c_nH_n(X_i)}{\sqrt{n!}}\right],$$

where $H_n(X_i)$ is a Hermite polynomial, $f_0(X_i)$ is a normal law of distribution.

The coefficients c_n are equal to zero for the normal law of distribution and characterize deviations of any other law of distribution from the normal.

The quantities are

$$\gamma_1 = c_3\sqrt{3!} = \frac{\mu_3}{\sigma^3},$$
$$\gamma_2 = c_4\sqrt{4!} = \frac{\mu_4}{\sigma^4} - 3,$$

where μ_k--central moments of random magnitude on the order of k; σ--mean quadratic deviation, called appropriately the coefficient of asymmetry and the coefficient of excess of distribution $f[X_i]$.

We must note that in our case we are examining the law of distribution $f[X_i(t)]$ and consequently, all quantities: γ_1 and γ_2 and σ_i are functions of time.

The problem of defining estimates for γ_1 and γ_2 for all compoments of motion was solved simultaneously with the problem of defining estimates of mathematical expectations and the elements of the covariation matrix by the Monte Carlo method. Because for each fixed moment in time a selection of random quantities was developed by volume N, the dispersions of estimates of the coefficients of asymmetry γ_1 and excess γ_2, according to [6], are equal to the following quantities:

$$D(\gamma_1) = \frac{6(N-1)}{(N+1)(N+3)},$$
$$D(\gamma_2) = \frac{24N(N-2)(N-3)}{(N+1)^2(N+3)(N+5)}.$$

There is a rather simple criterion of concord [6] of this law of distribution with the normal law: if the selective coefficients of asymmetry and excess satisfy the inequalities /102

$$|\gamma_1| \leqslant 3\sqrt{D(\gamma_1)},$$
$$|\gamma_2| \leqslant 5\sqrt{D(\gamma_2)},$$

then the observable distribution can be considered normal.

In the phase of oriented motion, the processing of results obtained by the Monte Carlo method was brought to 30 versions. Where N = 30, $3\sqrt{D(\gamma_1)} = 1.23$, $5\sqrt{D(\gamma_2)} = 3.5$.

For an AES without a passive stabilization system, in the phase of oriented motion the following maximum values were obtained for the absolute values of estimates γ_1 and γ_2:

$$\max|\gamma_1\omega_i|=1{,}17, \quad \max|\gamma_2\omega_i|=2{,}3, \quad i=x,y,z,$$
$$\max|\gamma_1\theta|=\max|\gamma_1\varphi|=1{,}03, \quad \max|\gamma_2\theta|=\max|\gamma_2\varphi|=1{,}6.$$

For an AES with magnetic damper:

$$\max|\gamma_1\omega_i|=3{,}4, \quad \max|\gamma_2\omega_i|=0{,}9\cdot 10^2,$$
$$\max|\gamma_1\theta|=\max|\gamma_1\varphi|=0{,}84, \quad \max|\gamma_2\theta|=\max|\gamma_2\varphi|=0{,}5\cdot 10^2$$

Similar estimates for coefficients of asymmetry and excess were obtained also for all other phases of motion of the AES about the center of mass.

Therefore, with motion along a circular orbit of a dynamically symmetrical AES without any passive stabilization system, the "normal" response in variation of orientation angles and projections of angular velocity of rotation with a rather simple correlation matrix is produced by "normal" perturbations in initial angular velocity of rotation. The presence on board the AES of a passive stabilization system alters the nature of system response: the laws of distribution of components of motion sharply differ from normal, the correlation matrix of motion varies in time and the nature of these changes is quite complicated.

Consequently, the present on board the AES of a system of passive stabilization complicates correlation analysis of motion.

But we must against stress that the system of magnetic damping effectively facilitates satellite orientation in space in a specific manner and suppression of random perturbations of motion.

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